

Expect FOURTEEN Plane Geometry problems on your ACT Math Test.

Disclaimers: Even though it is called a "cheat" sheet, do not use this sheet during your actual test! Keep in mind that this page does not cover every concept you may need on the ACT Math Test. Also, this page may contain errors. If you find an error, we'd love to hear about it: service@MathOnTime.com

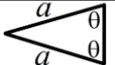
TRIANGLES

For ALL TRIANGLES, the 3 angles add to 180° and Area A=(1/2) base X height

Equilateral Triangle has 3 identical sides and 3 identical angles of 60° (since 60°+60°+60°=180°).



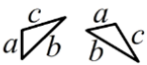
Isosceles Triangle has 2 identical sides & 2 identical angles, as shown.



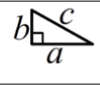
30°-60°-90° Right Triangle has sides with ratio 1 : 2 : sqrt(3) (hypotenuse = 2)



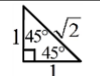
Congruent Triangles have identical side lengths and angles. They may be rotated or reflected relative to one another and still be congruent.



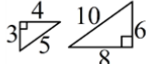
A Right Triangle has one 90° angle, sides that satisfy a^2+b^2=c^2 (Pythag. Thm.), & the other 2 angles are <90°



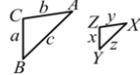
45°-45°-90° Right Triangle has sides with ratio 1 : 1 : sqrt(2)



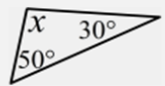
3 - 4 - 5 Right Triangle has sides proportional to 3, 4 & 5.



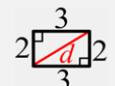
Similar Triangles have identical corresponding angles, & proportional side lengths, a/x = b/y = c/z



Ex. Find x in degrees. The 3 angles add to 180°, so x + 50° + 30° = 180° x = 180° - 50° - 30° = 100°

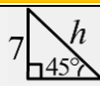


Ex. A rectangle has sides of length 3 and 2 inches. What is the diameter of the rectangle? The diameter divides the rectangle into two right triangles with legs 2 & 3, so d = sqrt(3^2 + 2^2) = sqrt(13) approx 3.6 in.

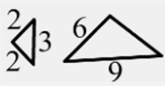


Ex. If a right triangle has hypotenuse 15 mm and one leg 12 mm, find the length of the other leg. Pythagorean thm. says 12^2 + x^2 = 15^2 So x = sqrt(225 - 144) = 9 mm.

Ex. Find h for the right triangle shown. The angles must add to 180° so this is a 45-45-90 right triangle. Thus the sides are in the ratio 1:1: sqrt(2), or 7:7: 7sqrt(2), so h = 7sqrt(2).

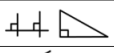


Ex. The two triangles shown are similar triangles. What is the perimeter of the larger triangle? The side of length 9 corresponds to the side of length 3, so the side of length 6 and the unknown side must correspond to the two sides labeled 2. Thus the unknown side is 6 and P = 6+6+9 = 21.



ANGLES

Right Angles measure 90° Right angles are indicated by a little square:



Supplementary angles add to 180° Ex. x + 148° = 180°, so x = 32°



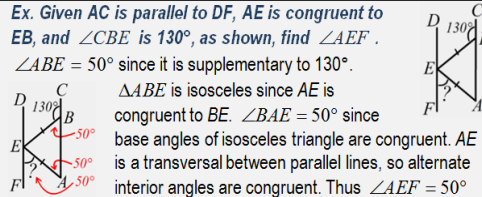
Vertical angles are congruent Ex. theta = 85°



For parallel lines cut by a transversal, alternate interior angles are congruent. Ex. theta = 50°



Ex. Given AC is parallel to DF, AE is congruent to EB, and angle CBE is 130°, as shown, find angle AEF. angle ABE = 50° since it is supplementary to 130°. Triangle ABE is isosceles since AE is congruent to BE. angle BAE = 50° since base angles of isosceles triangle are congruent. AE is a transversal between parallel lines, so alternate interior angles are congruent. Thus angle AEF = 50°



CIRCLES

Area A = pi r^2, Circumference = 2 pi r, diam. d = 2r

The perpendicular bisector of a chord in a circle passes through the center of the circle.



Measure of arc AB = x° Length of arc AB = 2 pi r (x / 360°)



Ex. A circle has diameter 5 cm. Find the area. A = pi r^2 and d = 2r, so A = pi (5/2)^2 = 25 pi / 4 cm^2.

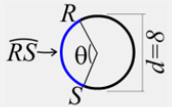
Ex. A chord in a circle has length 8" and is 3" from the center of the circle. Find the radius of the circle.



The line segment labeled 3" is perpendicular to the chord, so it bisects it. A right triangle is formed with the hypotenuse equal to the radius. This is a 3-4-5 right triangle, so r = 5 in.



Ex. A circle has diameter d=8 ft and theta = 120°. Find the length of arc RS. The radius is r = d / 2 = 4 ft, so RS = 2 pi r (theta / 360°) = 2 pi (4) (120° / 360°) = 8 pi / 3 approx 8.4 ft.



POLYGONS

Interior angles of n-sided polygon add up to (n - 2) 180°.

Perimeter of a polygon is the sum of the side lengths.

A Regular polygon has equal side lengths and equal angles each measuring (n - 2) 180° / n



A Quadrilateral is a 4-sided polygon. The angles of a quadrilateral sum to (4 - 2) 180° = 360°



A Trapezoid is a quadrilateral with one pair of parallel sides. Find area by dividing into simpler shapes.



A Parallelogram is a quadrilateral with two pairs of parallel sides. Opposite sides are equal, opposite angles have equal measure.



A Regular Pentagon is a 5-sided polygon with equal sides and angles. The five angles each measure (5-2)180°/5=108°.



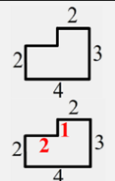
Square with side length s has Area = s^2, Perimeter = 4s



A Rectangle has Area = l · w, Perimeter = 2l+2w



Ex. Find the perimeter of the polygon given that all sides meet at 90°. The unlabeled lengths are found by making sure the total length on one side of the figure adds up to the total length on the opposite side, (2+2=4 and 2+1=3). Perimeter = 3+4+2+2+1+2 = 14



Ex. Three of the (interior) angles of a quadrilateral are 75°, 45°, and 125°. Find the fourth interior angle. In a quadrilateral, the four angles must add to 360°, so the 4th angle is 360° - 75° - 45° - 125° = 115°

Ex. A parallelogram has one interior angle of 100°. Find the other 3 interior angles. For a parallelogram, the 4 angles add to 360° and opposite angles have the same measure. Thus the angle opposite the 100° angle is also 100°, and the other two are both theta where theta + theta + 100° + 100° = 360°, or theta = 160° / 2 = 80°. Answer: 100°, 80°, 80°

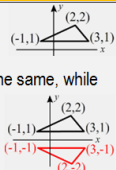
TRANSLATION, ROTATIONS, REFLECTIONS

A translation slides an object to another location (no rotation).

A reflection flips an object across a line (often x or y-axis) to create a mirror image.

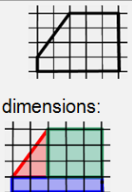
A rotation rotates an object about a point (often the origin).

Ex. This triangle is reflected across the x-axis to form a new triangle. Find the three corner points of the new triangle. The x-coordinates of all three points remain the same, while the y-coordinate changes sign. (think about Quadrants.) Thus the point (3,1) becomes (3, -1) when reflected, and the other two are (-1, -1) and (2, -2).



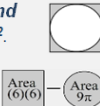
VOLUME AND AREA EXAMPLES

Ex. A shape is drawn on a grid with unit squares. Find the area in square units. Divide the area into simpler shapes & count units along edges of each shape for dimensions: Area = triangle + square + rectangle = 1/2 (2 x 3) + (3 x 3) + (5 x 1) = 17 square units



Ex. If one square tile is needed to cover an 8" X 8" area, how many are needed to cover a 10' by 20' floor? Convert floor dimensions to inches by multiplying by 12 to get 120" X 240". Along the 120" side, 120/8=15 tiles are needed, and along the 240" side, 240/8=30 tiles are needed. So covering the rectangular floor requires a total of 15 X 30 or 450 tiles.

Ex. A circle is inscribed inside a square. Find the shaded area if the circle has area 9 pi m^2. Since the area of the circle is pi r^2=9 pi, the radius of the circle is r=3 m. The side length of the square is the diameter of the circle, s=2r=6 m. Thus the square has area 6X6 = 36 m^2. The shaded area is the area of the square minus the area of the circle, 36 - 9 pi m^2.



Ex. Two identical cubes with 2 ft edges are stacked. Find the surface area and volume of the resulting figure. The front, back, and two sides are identical with surface areas 4'X2'. The top and bottom have surface area 2'X2'. Total surface area is the sum of the 6 sides, 4 (4 x 2') + 2 (2 x 2') = 32 + 8 = 40 ft^2 The volume is V = l · w · h = 4 · 2 · 2 = 16 ft^3



Ex. A box-shaped aquarium with a height of 20 inches holds 6000 cubic inches of water when filled to the top. How much water does it hold when filled to a height of just 12 inches? The volume of water held by the aquarium is V = l · w · h, but the length and width are the same whether the tank is filled to 20 or 12 inches. Thus the following proportion will give the desired volume: 12 is to 20 as V is to 6000 or 12 / 20 = V / 6000 Solving gives V = 12(6000) / 20 = 3600 in^3

